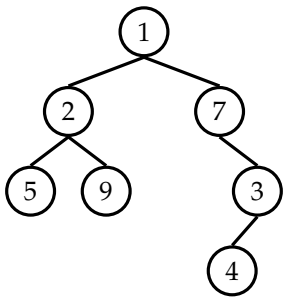
CSM 61B Spring 2017

Binary Trees

Week 9: March 13, 2017

1 Binary Trees

```
public class BinaryTree<T> {
    protected Node root;
    protected class Node {
        public T value;
        public Node left;
        public Node right;
    }
}
```



1.1 Define a procedure, `height`, which takes in a `Node` and outputs the height of the tree. Recall that the height of a leaf node is 0.

```
private int height(Node node) {
    if (node == null) {
        return -1;
    }
    return 1 + Math.max(height(node.left), height(node.right));
}
```

What is the runtime of `height`?

$\Theta(N)$, where N is the number of nodes in the tree.

1.2 Define a procedure, `isBalanced`, which takes a `Node` and outputs whether or not the tree is balanced. A tree is **balanced** if the left and right branches differ in height by at most one and are themselves balanced.

```
private boolean isBalanced(Node node) {
    if (node == null) {
        return true;
    } else if (Math.abs(height(node.left) - height(node.right)) <= 1) {
        return isBalanced(node.left) && isBalanced(node.right);
    }
    return false;
}
```

What is the runtime of `isBalanced`?

$\Theta(N)$ in the best case, $\Theta(N \log N)$ in the worst case, and $\Omega(N), O(N \log N)$ overall.

2 Binary Search Trees

2.1 Write the `fromSortedArray` method for binary search trees. Given a sorted `int[]` `array`, efficiently construct a balanced binary search tree containing every element of the array.

```
public class BinarySearchTree<T extends Comparable<T>> {
    protected Node root;
    protected class Node {
        public T value;
        public Node left;
        public Node right;
    }
    public static BinarySearchTree<Integer> fromSortedArray(int[] values) {
        BinarySearchTree<Integer> bst = new BinarySearchTree<>();
        bst.root = bst.fromSortedArray(values, 0, values.length - 1);
        return bst;
    }
    private Node fromSortedArray(int[] values, int lower, int upper) {
       if (lower > upper) {
           return null;
       }
       int middle = lower + ((upper - lower) / 2);
       Node mid = new Node();
       mid.value = values[middle];
       mid.left = fromSortedArray(values, lower, middle - 1);
       mid.right = fromSortedArray(values, middle + 1, upper);
       return mid;
    }
}
```

3 Balanced Faster *Extra for Experts*

3.1 Design an algorithm to more efficiently compute whether a tree is balanced or not. *Hint: What part of the naive* `isBalanced` *solution was expensive and how can we make it less expensive?*

Since the `height` calls are expensive, use an `int` return type to carry more information about the state of the tree when performing the recursive step.

```
private boolean isBalanced(Node node) {
    return isBalancedFast(node) > -2;
}
private int isBalancedFast(Node node) {
    if (node == null) {
        return -1;
    }
    int leftResult = isBalancedFast(node.left);
    int rightResult = isBalancedFast(node.right);
    if (leftResult < -1 || rightResult < -1 || Math.abs(leftResult - rightResult) > 1) {
        return -2;
    }
    return 1 + Math.max(leftResult, rightResult);
}
```

4 Successor *Extra for Experts*

4.1 Given a node in a binary search tree (with parent pointers), write a successor method which returns the next node in the in-order traversal of the BST. If there is no successor, simply return null.

```
public class BinarySearchTree<T extends Comparable<T>> {
    protected Node root;
    protected class Node {
        public T value;
        public Node parent;
        public Node left;
        public Node right;
    }
    private Node successor(Node node) {
        if (node.right != null) {
            node = node.right;
            while (node.left != null) {
                node = node.left;
            }
            return node;
        } else {
            Node parent = node.parent;
            while (parent != null && parent.right == node) {
                node = parent;
                parent = parent.parent;
            }
            return parent;
        }
    }
}
```