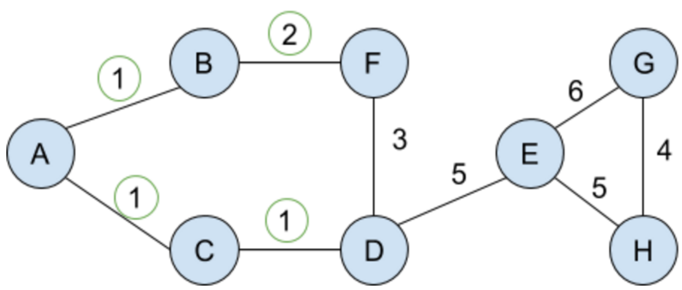
CS 61B Spring 2017 Guerrilla Section 6 Solution

15 April 2017

Directions: In groups of 4-5, work on the following exercises. Do not proceed to the next exercise until everyone in your group has the answer and *understands why the answer is what it is.* Of course, a topic appearing on this worksheet does not imply that the topic will appear on the midterm, nor does a topic not appearing on this worksheet imply that the topic will not appear on the midterm.

1 MSTs

(1)



Consider the undirected graph above. We are trying to find the minimum spanning tree (MST) of the graph. The edges with their weight labels circled have already been added to our MST.

(a) What is the next edge to be added to our MST if we are using Kruskal's Algorithm?
(G, H). Kruskal's algorithm adds the lightest edge that does not cause a cycle. The lightest edge is (D, F), but this causes a cycle. The next lightest edge is (G, H). This does not cause a cycle so we add it to our MST.

(b) What is the next edge to be added to our MST if we are using Prim's Algorithm?
Note: part (a) and part (b) are not related. Don't consider the edge that you added in part (a).
(D, E). Prim's algorithm adds the lightest edge that is connected to the current tree, but does not cause a cycle. The lightest edge connected to the current tree is (D, F), but this causes a cycle. The next lightest edge connected to the current tree is (D, E) which does not cause a cycle so we add it to our MST.

(c) What is the weight of the complete MST?
19. Added edges: $(G, H), (D, E), (E, H)$. The total weight is $1 + 1 + 1 + 2 + 4 + 5 + 5 = 19$. Note that both Prim's Algorithm and Kruskal's Algorithm always MSTs of the same total weight (the minimum total weight).

(2) Consider a graph with negative edges.

(a) How would we modify Kruskal's Algorithm to find a MST on this graph?
No modifications necessary. The negative edges have no effect on Kruskal's Algorithm.

(b) We now want to find a minimum spanning graph (no longer needs to be a tree) for this graph. How would we modify Kruskal?s Algorithm to find the minimum spanning graph for this graph?
We could run Kruskal's Algorithm, and then add all remaining negative edges. This would ensure that we have included all of the lightest edges and all of the negative edges, since the negative edges decrease the weight of the minimum spanning graph.

2 To Cycle Or Not To Cycle

(1) Given an undirected graph $G = (V, E)$ and an edge $e = (s, t)$ in G, create an $O(V + E)$ time algorithm to determine whether G has a cycle containing e. No code needed. Just describe.
Remove e from the graph. Then DFS or BFS starting at s to find t.

(2) (Extra for Experts. Skip this and come back if you have time)
Given a connected, undirected, weighted, graph, describe an algorithm to construct a set with as few edges as possible such that if those edges were removed, there would be no cycles in the remaining graph. Additionally, choose edges such that the sum of the weights of the edges you remove is minimized. This algorithm must be as fast as possible.
Negate all edges.
Form an MST via Kruskal's/Prim's algorithm.
Return the set of all edges not in the MST (undo negation).

3 Start To Finish

You're given an undirected, positively weighted graph $G = (V, E)$, a list of start vertices S, and a list of end vertices T. Describe an efficient algorithm that returns the shortest path, such that the path starts at one vertex from S and ends at one vertex from T.

Hint: Consider adding dummy nodes to the graph to reduce this problem into something simpler.
Add a node S, connected to all start nodes and a node E connected to all end nodes with weight C (any non-negative constant). Find the shortest path from S to E, remove S and E from the final problem. This can be done with Dijkstra's algorithm.

4 One Path to Traverse them All, and Topological Sort Them

Given a directed acyclic graph G, write an algorithm that determines if G contains a path that goes through every vertex exactly once. Briefly justify why the algorithm is correct, and state the runtime.

Assume that the graph is implemented with the following API, where nodes are represented by integers.

```
public class Graph {
  // Returns true if this graph has an edge from u to v.
  public boolean hasEdge(int u, int v);

  // Returns a list of integers, in a topologically sorted order for this graph;
  // implemented in the way described in lecture.
  public List<Integer> topologicalOrder();
}

// Please implement your algorithm in this method:
public boolean onePath(Graph g) {
  /* The solution */
  List<Integer> topoSorted = g.topologicalOrder();
  for (int i = 0; i < topoSorted.length - 2; i++) {
    if (!g.hasEdge(topoSorted[i], topoSorted[i+1])
      return false;
  }
  return true;
  /* End solution */
}
```

Justification: The main idea/justification is that if a DAG has a path that traverses every vertex, then its topological sorting must be unique, and additionally, every pair of nodes in the topological sorting must have an edge between them.

Runtime (in Θ notation): $\Theta(|V| + |E|)$, the runtime of topological sorting.

STOP!

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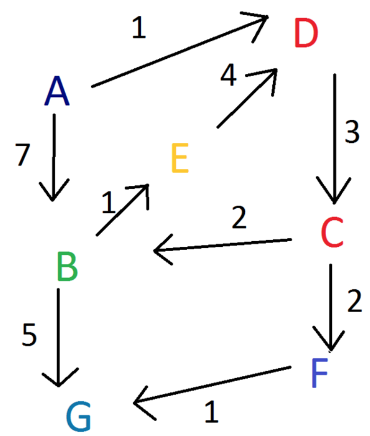
5 The Amazing Race!

Directions: With the United States as your starting point, your goal is to travel to Greece as fast as you can. However, there is a twist such that you can only travel the paths shown on the graph toward Greece. (Note: This graph was not designed by a geographer.)



Letter in graph	Name of location	Heuristic to End
A	America (USA)	6
B	Brazil	4
C	China	7
D	Dominican Republic	7
E	Egypt	2
F	France	6
G	Greece	0

(a) Run Dijkstra's algorithm to find the shortest paths to all destinations starting from A, breaking ties alphabetically if necessary.

$$A : c = 0$$
$$D : c = 1$$
$$C : c = 4$$
$$B : c = 6$$
$$F : c = 6$$
$$E : c = 7$$
$$G : c = 7$$

Dijkstra's keeps track of a priority queue, popping off the node with the shortest distance from the start in each iteration.

(b) Use A* search to find the optimal path to Greece, breaking ties alphabetically if necessary.

$$A : c = 6$$
$$A \rightarrow B : c = 11$$
$$A \rightarrow D : c = 8$$
$$A \rightarrow D \rightarrow C : c = 11$$
$$A \rightarrow B \rightarrow E : c = 10$$
$$A \rightarrow B \rightarrow G : c = 12$$
$$A \rightarrow B \rightarrow E \rightarrow D : c = 19$$
$$A \rightarrow D \rightarrow C \rightarrow B : c = 10$$
$$A \rightarrow D \rightarrow C \rightarrow F : c = 12$$
$$A \rightarrow D \rightarrow C \rightarrow B \rightarrow G : c = 11$$

$ADCBG$ is the solution based on A* search. This is not the optimal solution; the optimal solution is $ADCFG$ with a cost of 7. A* search gave a suboptimal solution because F has an inadmissible heuristic. (The optimal path from F to G is 1, but the heuristic values given provided a heuristic value of 6 for F).

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6 Which Sort to Use?

For each of the following scenarios, choose the best sort to use and explain your reasoning.

(a) The list you have to sort was created by taking a sorted list and swapping N pairs of adjacent elements.
Insertion sort, since a list created in such a manner will have at most N inversions. Recall that insertion sort runs in $\Theta(N + K)$ time, where K is the number of inversions.

(b) You have to sort a list on a machine where swapping two elements is much more costly than comparing two elements (and you want to do the sort in place).
Selection sort, since in its most common implementation, selection sort performs N swaps in the worst case, whereas all other common sorts perform $\Omega(N \log N)$ swaps.

(c) Your list is so large that not all of the data will fit into RAM at once. As is, at any given time most of the list must be stored in external memory (on disk), where accessing it is extremely slow.
Merge sort is ideal here, since its divide-and-conquer strategy works well with the restriction on only being able to hold a partition of the list in RAM at any given time. Sorted runs of the list can be merged in RAM and flushed to disk one block at a time, minimizing disk reads and writes.

(d) Given a list of emails ordered by send time, sort the list such the emails are ordered by the sender's name first while secondarily maintaining the time-ordering.
Merge sort. We should avoid quicksort since the usual implementation is unstable and can change the time-ordering when sorting.

(e) You have a randomly shuffled list of numbers unbounded in size and want to sort the elements.
Quicksort has better real-world characteristics than merge sort in this situation and we don't need to consider stability either.

(f) Suppose you're designing a secure system that needs to defend against *adversarial inputs*. An attacker can give you any list they choose and understand exactly how your sorting algorithms are implemented.
Merge sort or heap sort have worst-case time complexity in $\Theta(N \log N)$. Quicksort with random pivots would also be acceptable since it's highly improbable to repeatedly select poor pivots on large inputs. Selection sort, or insertion sort would not be acceptable here because of their worst-case $\Theta(N^2)$ runtime.

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7 Empirical Analysis

Andrew, Kevin, and Fahad performed timing tests on several sorting algorithms: selection sort, insertion sort, merge sort, and tree sort (repeated insertions into a binary search tree with no attempt to balance, followed by a traversal of the tree). They timed each sorting algorithm on several datasets of 2000 values. Unfortunately, Andrew forgot to label each experiment with its sort! Help them out by figuring out which times match with which sorting method.

Time to sort 2000 random values	Time to sort 2000 values already in increasing order	Time to sort 2000 values already in decreasing order	Sorting method (Selection, Insertion, Merge, or Tree)
1098	29	1685	Insertion
183	1624	1570	Tree
191	207	195	Merge
1698	1776	1734	Selection

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8 What's that Sort!?

Which sorting algorithms do the following illustrate? Your options are merge sort, insertion sort, selection sort, heap sort, quick sort. Algorithms illustrated may not conform exactly to those presented in discussion and in lecture. Please note that each of these are snapshots as the algorithm runs, not all iterations of its running.

(a)
```
5103 9914 0608 3715 6035 2261 9797 7188 1163 4411
0608 1163 5103 3715 6035 2261 9797 7188 9914 4411
0608 1163 2261 3715 6035 5103 9797 7188 9914 4411
```

Selection sort

(b)
```
5103 9797 0608 3715 6035 2261 9914 7188 1163 4411
0608 3715 2261 1163 4411 5103 9797 6035 9914 7188
0608 3715 2261 1163 4411 5103 6035 7188 9797 9914
```

Quicksort

(c)
```
dze ccf hwy pjk bkw xce aux qtr
ccf dze hwy pjk bkw xce aux qtr
ccf dze hwy pjk aux bkw qtr xce
aux bkw ccf dze hwy pjk qtr xce
```

Merge sort

(d)
```
dze ccf bkw hwy pjk xce aux qtr xpa atm
dze ccf bkw hwy pjk xce aux qtr atm xpa
dze ccf bkw hwy pjk xce aux atm qtr xpa
dze ccf bkw hwy pjk xce atm aux qtr xpa
dze ccf bkw hwy pjk atm aux qtr xce xpa
dze ccf bkw hwy atm aux pjk qtr xce xpa
dze ccf bkw atm aux hwy pjk qtr xce xpa
dze ccf atm aux bkw hwy pjk qtr xce xpa
dze atm aux bkw ccf hwy pjk qtr xce xpa
atm aux bkw ccf dze hwy pjk qtr xce xpa
```

Insertion sort (from the right)

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